A Non-Fickian Mixing Model for Stratified Turbulent Flows

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LONG-TERM GOALS

The long term goal of this project is to develop a better understanding of oceanic processes in the range of 100 m to 10 km, in the so-called sub-meso-scale range. In particular, it is important to explore and find out whether and what type of sub-meso-scale instabilities exist, how they are connected to both larger scale and smaller scale motions, and to what extent they influence transport processes in the ocean. Another important objective of this project is to test how well subgrid-scale (SGS) models for large eddy simulations (LES) work in the presence of backward energy cascade that may be characteristic in sub-meso-scale motions. Another long term objective of this effort would be how to improve the predictive skill of the Navy numerical models in the light of the impact of sub-mesoscale motions in the ocean.

OBJECTIVES

My main objective in this first phase of the project is to model mixed layer instabilities, investigate their behavior and try to develop sampling strategies using synthetic drifters and tracers, especially considering potential limitations of resources during the upcoming experimental phase of the Lateral Mixing DRI.

APPROACH

The work is based on large eddy simulations using the non-hydrostatic spectral element model Nek5000 (Fischer, 1997).

WORK COMPLETED

A set of 99 LES experiments have been conducted in order to develop a better understanding of the physical characteristics of mixed layer instabilities. About half of these experiments are been devoted to test sampling strategies with synthetic drifters and tracers in preparation for the upcoming field experiments.

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RESULTS

1) Sampling of mixed-layer instability using Lagrangian particles:

The experimental configuration for the generation of mixed-layer instability using LES was described in the annual report of 2009. Over the past year, I have focused entirely on sampling strategies using two categories of fields. The first one emerges in strongly-stratified fronts, and it is characterized by pan-cake type, coherent sub-mesoscale eddies, of O(2) km diameter, which induce strong lateral stirring (Fig. 1, left panel). The other regime appears in the limit of weak stratification and weak rotation, where the Burger number scaling and wavenumber spectrum of kinetic energy density indicate much smaller size eddies, of O(200) m diameter, that have strong vertical coherence (Fig. 1, right panel).

Both types of fields are sampled by launching 5625 particles (625 triplets released at three depths levels; near the surface, z_0=5 m, in the middle of the mixed layer, z_0=50 m, and at the mixed layer base, z_0=100 m). Lagrangian particles are advected using two methods: full 3D advection with (u,v,w), and also 2D advection with (u,v) at z_0=5 m. The main justification for this approach is that while 3D advection is fully Lagrangian, there are not many drifters available in large enough numbers to document most degrees of freedom. Our main metric is the scale-dependent finite-scale Lyapunov exponent (FSLE) which quantifies relative dispersion as a function of drifter pair separation. FSLE is therefore ideally suited to explore multi-scale behavior of lateral stirring under the action of turbulent processes.

Effect of stratification on the FSLE: It is found that when the fluid is strongly-stratified, relative dispersion is exponential, or approximately scale-independent below the size of the sub-mesocale eddies (Fig. 2, left panel). In the weakly-stratified regime, we find a scaling consistent with the Richardson regime for separations greater than size of the sub-mesoscale eddies. It is also shown that the limiting values of the FSLEs collapse under the scaling proposed by Poje et al. (2010), namely using the positive/hyperbolic values of the Okubo-Weiss partition.

Depth-dependence of the FSLE: Depth dependence of the FSLE is also explored (Fig. 2, right panel) and the maximum FSLE value is shown by decrease by the factor of two between 5 m and 50 m depth, and another factor of two at 100 m depth release. Most interestingly, no major difference is found in the FSLEs computed from 3D and 2D advection, indicating that when particles are released at horizontal surfaces, their initial relative dispersion is dominated by horizontal processes (mostly by the high strain regions).

Sensitivity to the number of drifters: Possibly the most important question for sampling is the number of drifters needed in order to get reasonable estimates of the relative dispersion. The FSLE is computed by sub-sampled sets of particles consisting of 375, 96, 27 and 9 triplets, in comparison to the full set of 1875 triplets. This is carried out for both strongly and weakly-stratified frontal instability. We focus on 5-m releases since these are the most energetic. The results (Fig. 3) show that even with 9 triplets, the FSLE curves can be estimated in a fairly unbiased way, or within a factor of approximately two at each separation distance. This is an encouraging result for observational programs, as it is clearly not feasible to deploy thousands of drifters of present technology in a small part of the ocean without environmental and other implications.

Turbulent coherent structures: Next, we employed spatial maps of the FSLE in order to visualize turbulent coherent structures responsible for transport in the mixed layer instability problem. This is done on the basis of releasing and advecting 390,963 particles at 5 m depth in the flow fields (Fig. 4, left panel). FSLE maps are computed in 2D because the dynamical system theory for 3D is not yet well developed. Forward time integration emphasizes regions of concentration, typically associated with stable manifolds, which are barriers to transport. In the strongly-stratified case (Fig. 4, right panel), clear coherent structures are found, while the weakly-stratified case yields features resembling isotropic turbulence.

2) Sampling of mixed-layer instability using passive tracers:

It is also desirable to test sampling strategies using a passive scalar (or equivalently, tracers). This is not only because tracer releases are becoming an integral part of oceanic experiments, but also this is a promising area experiencing introduction of new methods for three-dimensional mapping and airborne laser scanning towards a synoptic sampling of upper ocean fields (Sundermeyer et al., 2007), and will be used in upcoming LatMix field experiments.

Specifically, we aimed at addressing the following questions:

(a) How can one identify the difference between strongly and weakly-stratified mixed-layer flows using a passive scalar?

(b) What would be a useful metric to quantify the temporal evolution of the passive scalar concentration?

(c) Are the results sensitive to the initial amount of the passive scalar?

(d) What is the impact of 3D motions on the tracer fields?

Of particular interest is the competition between chaotic advection by coherent structures that tends to stretch and fold the initially smooth tracer field to increasingly finer size, thereby increasing the spatial gradients in the tracer field, and diffusion that acts to reduce the tracer gradients. Here, we investigate whether the measure put forward by Sundaram et al. (2009) can be useful to quantify this competition. This measure can be written in 2D and 3D versions as:

$$\chi^2_{2D}(z_0,t) = \frac{\int |\nabla C(x,y,z_0,t)|^2 \, dA}{\int |C(x,y,z_0,t)|^2 \, dA} \quad \text{and} \quad \chi^2_{3D}(t) = \frac{\int |\nabla C(x,y,z,t)|^2 \, dV}{\int |C(x,y,z,t)|^2 \, dV} \ ,$$

where C is the tracer concentration found by integrating an advection-diffusion equation starting from two initial conditions shown in Fig. 5; one is a 50 m wide patch that is feasible in oceanic experiments (ic1, left panel), while the other is 500 m wide (ic2, right panel), which is well resolved in our LES computations. Both are launched in well-developed fields shown in Fig. 1. The resulting 3D tracer fields from strongly and weakly-stratified cases are plotted in Fig. 6.

The 2D tracer metric is applied to fields at 5 m depth (Fig. 7, upper panel). It shows a decay with ic1, indicating that diffusion dominates over chaotic advection for such a narrow initial patch, while the

opposite applies for ic2. After 50 eddy turn over time scales, all solutions converge (dependence on ics disappears).

We are also exploring time evolution of the 3D tracer metric as function of the stratification parameter and molecular diffusivity of the tracer (Fig. 7, bottom panel).

IMPACT/APPLICATIONS

The scales considered in this project represent the range of scale of navy operations and thus anomalous currents and perturbations in the acoustic and optical environment that can affect a variety of navy operations. Understanding the motion in this range of scales is therefore critical to help improve the predictive capability of the existing Navy models.

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PUBLICATIONS

Özgökmen, P.F. Fischer, A.C. Poje and A.C. Haza: Large eddy simulations of mixed layer instabilities and sampling strategies. Ocean Modelling [submitted, refereed].

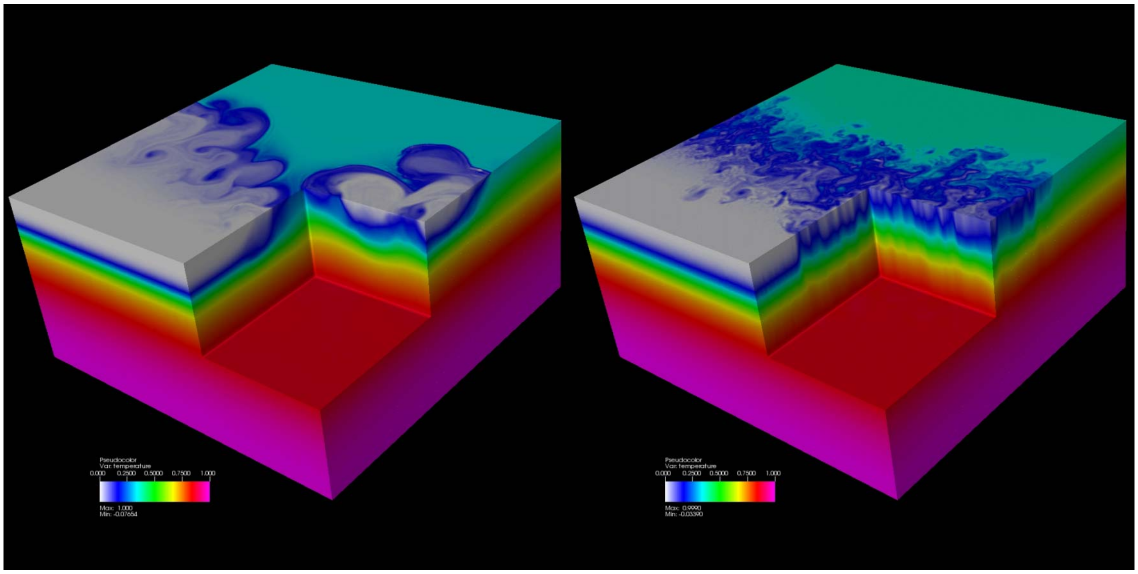

Fig. 1: Three-dimensional snapshot of the density perturbation fields in the mixed-layer instability for the strongly-stratified (Fr=0.1) case (left panel) and weakly stratified (Fr=1.0) case (right panel).

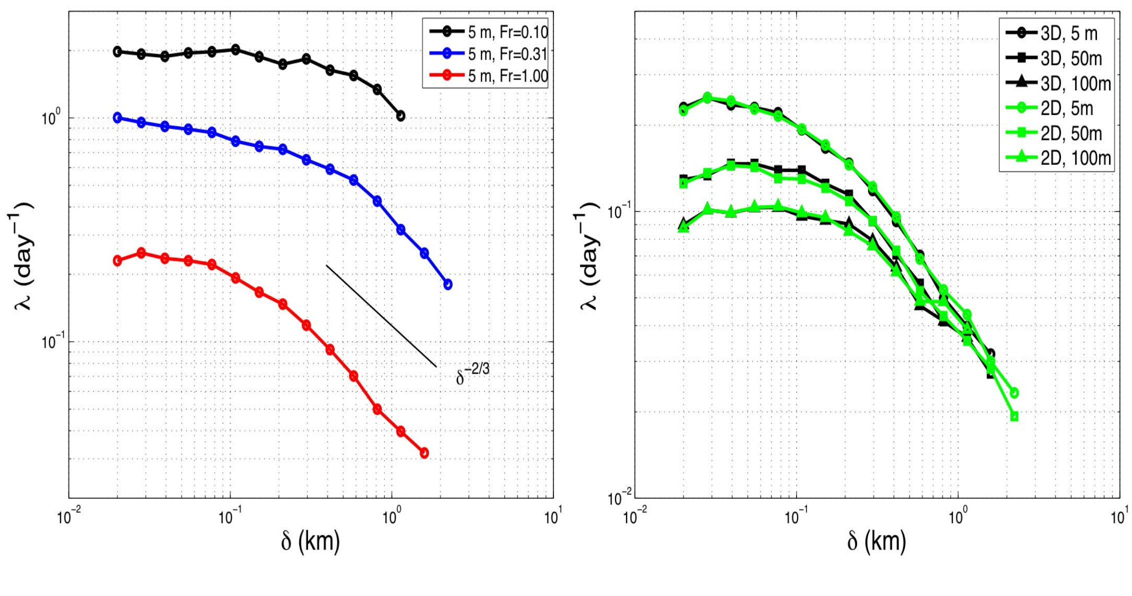

Fig. 2: (left panel) Dependency of the scale-dependent FSLE on the stratification parameter Fr from 5 m particle releases. The solid line indicates the slope of the Richardson's law dispersion. (Right panel) FSLE from releases at three depth levels as well as 3D (black) and 2D (green) particle advection for the weakly-stratified cases.

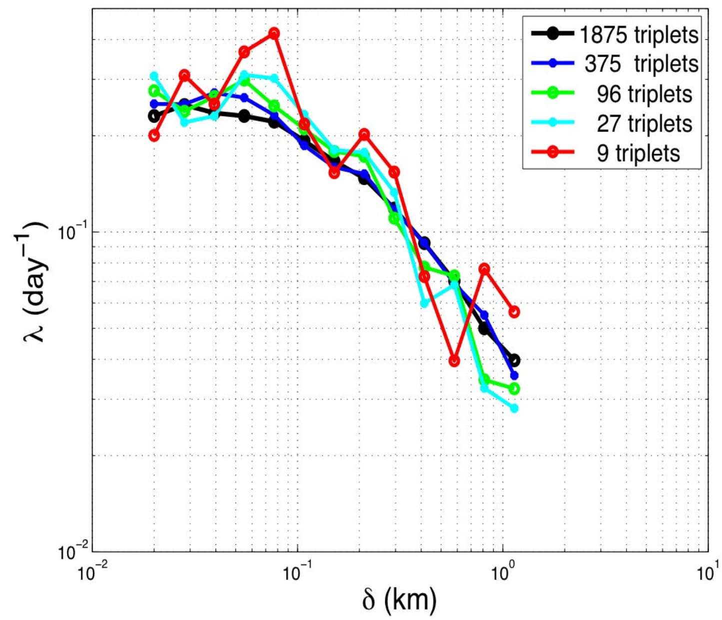

Fig. 3: Sensitivity of the FSLE to the number of drifter triplets in the weakly-stratified case.

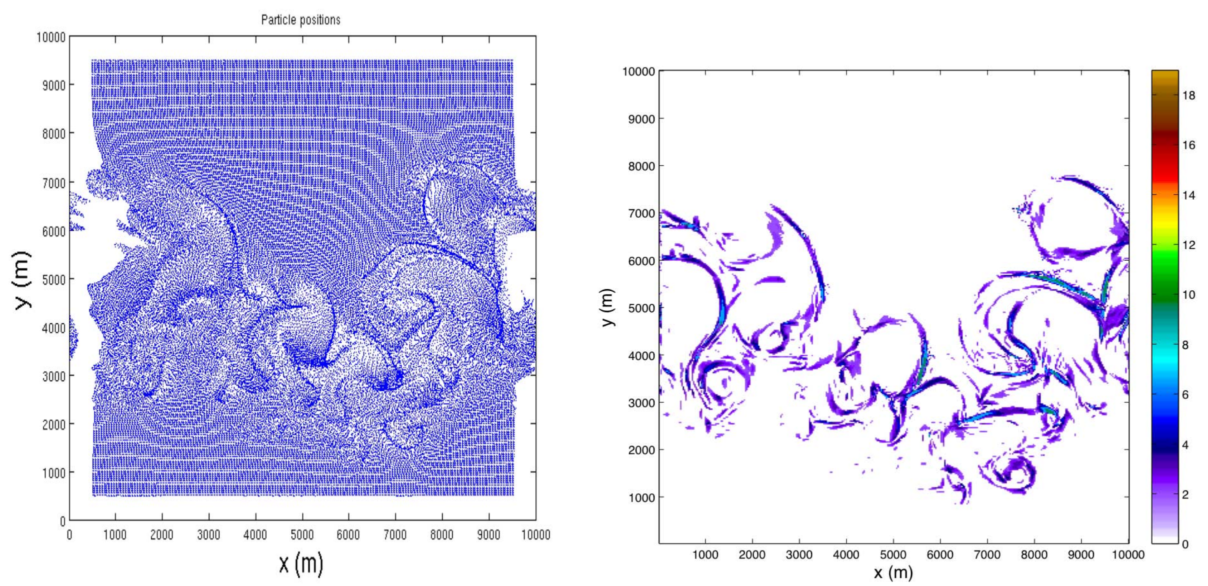

Fig. 4: (Left panel) Particle positions after about half a day advection.
(Right panel) FSLE map (color scale in 1/days) for the strongly-stratified case, corresponding to snapshot shown in the left panel of Fig. 1.

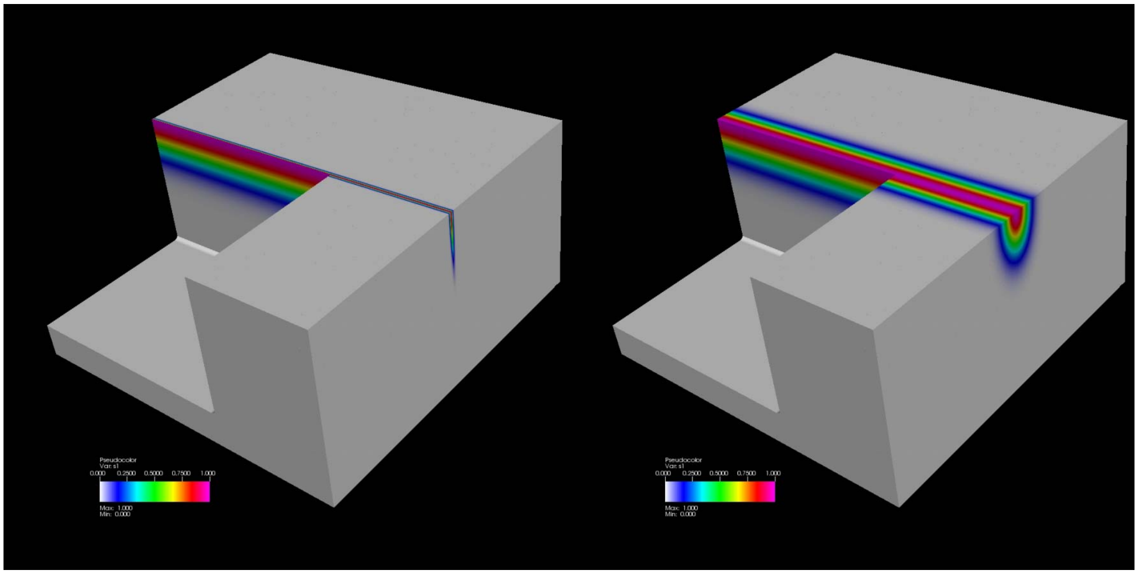

Fig. 5: Three-dimensional view of the initial tracer concentration field for (left panel) ic1 and (right panel) ic2.

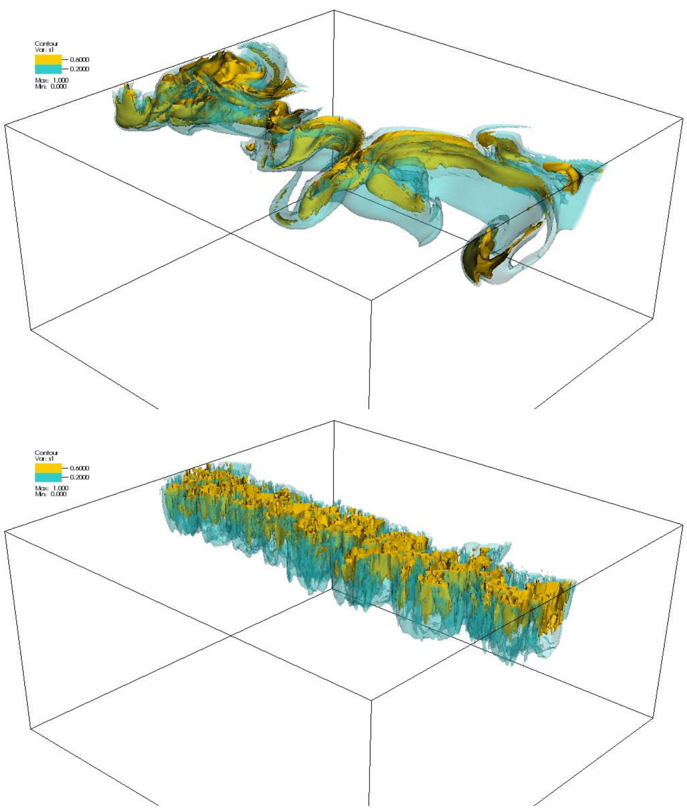

Fig. 6: Three-dimensional contour of tracer concentration with ic2 in the strongly-stratified (upper panel) and weakly-stratified (lower panel) cases.

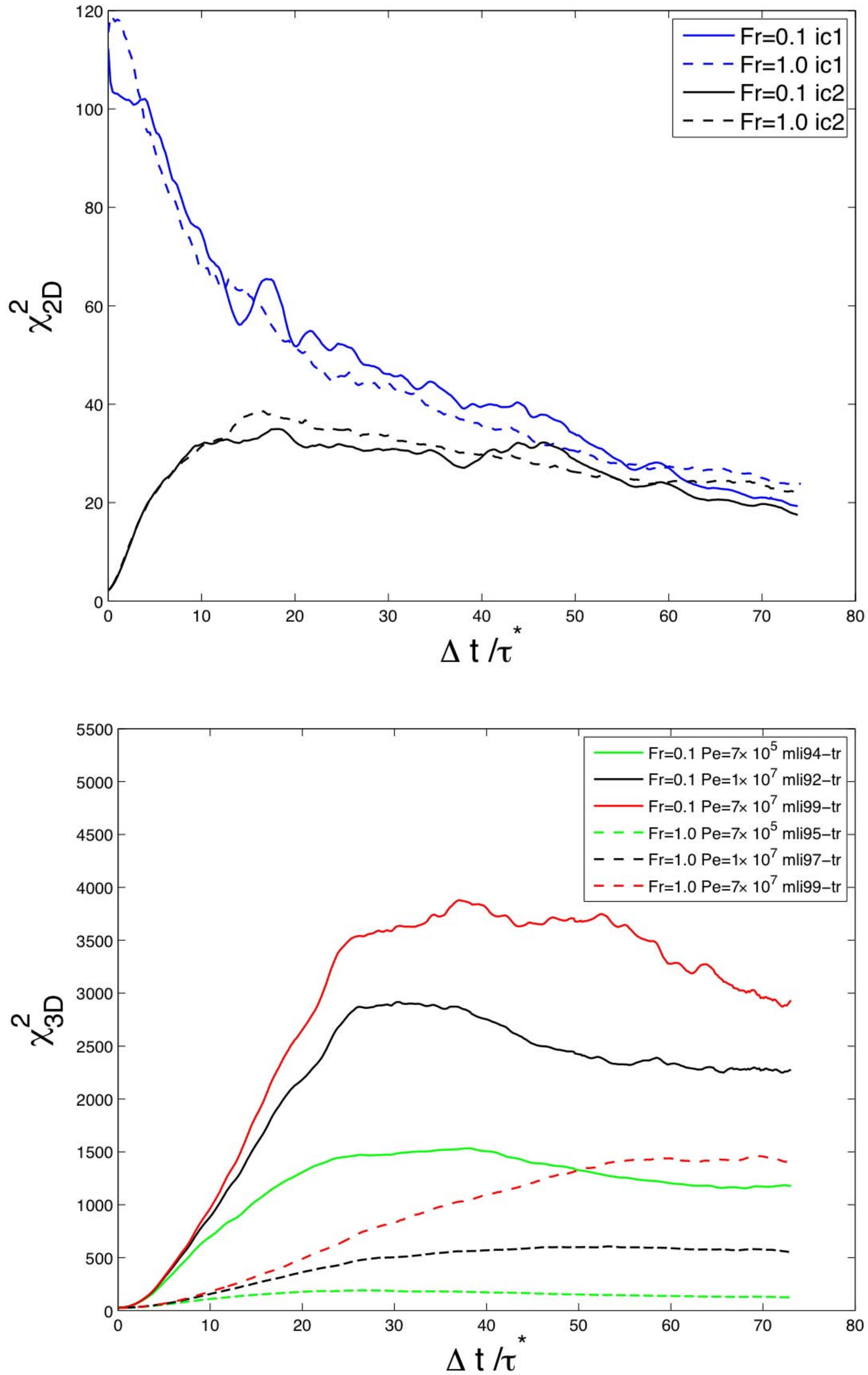

Fig. 7: (Upper panel) Comparison of 2D tracer dispersion metric in strongly and weakly stratified flows for the initial conditions shown in Fig. 5. (Lower panel) Evolution of the 3D tracer dispersion metric for different tracer diffusivities.